

$$\begin{aligned}
 \sin\left(z + \frac{\pi}{2}\right) &= \left\{ \sin z - \frac{e^{iz} - e^{-iz}}{2i} \right\} = \frac{e^{i\left(z + \frac{\pi}{2}\right)} - e^{-i\left(z + \frac{\pi}{2}\right)}}{2i} \\
 &= \frac{e^{iz} \cdot e^{\frac{\pi}{2}i} - e^{-iz} \cdot e^{-\frac{\pi}{2}i}}{2i} = \frac{e^{iz} \cdot (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) -}{2i} \\
 &- \frac{e^{-iz} \cdot (\cos \frac{\pi}{2} - i \sin \frac{\pi}{2})}{2i} = \frac{e^{iz} (0 + i) - e^{-iz} (0 - i)}{2i} \\
 &= \frac{ie^{iz} + ie^{-iz}}{2i} = \frac{i(e^{iz} + e^{-iz})}{2i} = \cos z
 \end{aligned}$$

$$\sin\left(\frac{\pi}{2} + z\right) = \cos z$$