

$$\operatorname{ctgh}(-\pi + 2i) = \left\{ \begin{array}{l} \operatorname{ctgh} z = \frac{\cos z}{\sin z} \\ \cos z = \frac{e^{iz} + e^{-iz}}{2} \\ \sin z = \frac{e^{iz} - e^{-iz}}{2i} \end{array} \right\} =$$

$$= \frac{e^{-\pi i - 2} + e^{\pi i + 2}}{2} \cdot \frac{2i}{e^{-\pi i - 2} - e^{\pi i + 2}} =$$

$$= \frac{e^{-2}(\cos \pi - i \sin \pi) + e^2(\cos \pi + i \sin \pi)}{2i}$$

$$= \frac{e^{-2}(-1 - 0) + e^2(-1 + 0)}{2i}$$

$$= \frac{-e^{-2} - e^2}{2i} \cdot \frac{(e^2 + e^{-2})}{2} =$$

$$\frac{-e^{-2} + e^2}{2i} \cdot i \frac{(e^2 - e^{-2})}{2} =$$

$$= \left\{ \begin{array}{l} \operatorname{cosh} x = \frac{e^x + e^{-x}}{2} \\ \operatorname{sinh} x = \frac{e^x - e^{-x}}{2} \end{array} \right\} = \frac{\operatorname{cosh} 2}{i \operatorname{sinh} 2} = -i \operatorname{ctgh} 2$$