$$f(z) = \cos z = \frac{e^{iz} + e^{-iz}}{2} = \frac{e^{i(x+yi)} + e^{-i(x+yi)}}{2} =$$

$$= \frac{e^{ix-y} + e^{-ix+y}}{2} = \frac{e^{-y}(\cos x + i\sin x)}{2}$$

$$+ \frac{e^{y}(\cos x - i\sin x)}{2} = \frac{e^{-y}\cos x}{2} + \frac{e^{-y} i \sin x}{2}$$

$$+ \frac{e^{y}\cos x}{2} - \frac{e^{y} i \sin x}{2} = \frac{\cos x}{2}\left(e^{-y} + e^{y}\right) +$$

$$+ i\frac{\sin x\left(e^{-y} - e^{y}\right)}{2} = \cos x \cdot \cosh y - i\sin x \sinh y$$

$\mathrm{Re}\, z = \cos x \cosh y \qquad \mathrm{Im}\, z = \sin x \sinh y$