$\cos z = -2i$

$\cos z = \frac{e^{iz} + e^{-iz}}{2}$

$\frac{e^{iz} + e^{-iz}}{2} = -2i$

$e^{iz} + e^{-iz} + 4i = 0$

$e^{iz} + \frac{1}{e^{iz}} + 4i = 0 \quad | \cdot e^{iz}$

$(e^{iz})^2 + 4ie^{iz} + 1 = 0$

$e^{iz} = t \qquad t^2 + 4it + 1 = 0$

$\Delta = \frac{-16-4}{~~~~} = -\frac{20}{~~~} = -20 \qquad \sqrt{\Delta} = \sqrt{-20} = 2\sqrt{5}i$

$t_1 = \frac{-4i - 2\sqrt{5}i}{2} = -2i - \sqrt{5}i$

$t_2 = -2i + \sqrt{5}i$

$e^{iz} = -2 - \sqrt{5}i$

$iz = \text{Log}(-2 - \sqrt{5}i)$

$\text{Log}\, z = \ln|z| + i\arg z$

$|z| = 2 + \sqrt{5} \qquad \cos\alpha = 0, \ \sin\alpha = -1 \ \Big\| \to \alpha = \frac{3}{2}\pi$

$\text{Log}\, z = \ln(2+\sqrt{5}) + i\frac{3}{2}\pi$

$iz = \ln(2+\sqrt{5}) + i\frac{3}{2}\pi$

$z_1 = -\frac{3}{2}\pi + i\ln(2+\sqrt{5})$

$e^{iz} = -2i+\sqrt{5}i$

$iz = \log(-2i+\sqrt{5}i)$

$\log(-2i+\sqrt{5}i) = \ln(-2+\sqrt{5}) + i\cdot\frac{\pi}{2}$

$iz = \ln(-2+\sqrt{5}) + i\cdot\frac{\pi}{2}$

$z = i\ln(-2+\sqrt{5}) - \frac{\pi}{2}$

$z_2 = -\frac{\pi}{2} + i\ln(-2+\sqrt{5})$

Odp. $z_1 = -\frac{3}{2}\pi + i\ln(2+\sqrt{5})$, $z_2 = -\frac{1}{2}\pi + i\ln(-2+\sqrt{5})$