$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \qquad z = x+yi$$

$$\frac{e^{(x+yi)i} - e^{-i(x+yi)}}{2i} = \frac{e^{xi-y} - e^{-ix+y}}{2i} =$$

$$= \frac{\left[e^{-y}(\cos x + i\sin x) - e^{y}(\cos x - i \sin x)\right]\cdot i}{2i \cdot i} =$$

$$= \frac{i e^{-y}(\cos x + i\sin x) - i e^{y}(\cos x - i\sin x)}{-2} =$$

$$= -\frac{\left(i e^{-y}\cos x - e^{-y}\sin x - i e^{y}\cos x - e^{y}\sin x\right)}{2}$$

$$= \frac{-i e^{-y}\cos x + e^{-y}\sin x + i e^{y}\cos x + e^{y}\sin x}{2} =$$

$$= \frac{e^{-y}\sin x + e^{y}\sin x}{2} + \frac{i\left(e^{y}\cos x - e^{-y}\cos x\right)}{2}$$

$$= \sin x \operatorname{ch} y + i \cos x \left(\frac{e^{y} - e^{-y}}{2}\right)$$

$$\sin x \operatorname{ch} y + \cos x \, \frac{e^{-y} - e^{y}}{2i}$$

$\operatorname{Re} z = \sin x \operatorname{ch} y \qquad \operatorname{Im} z = \cos x \operatorname{sh} y$