

$$|f(z)| = 1$$

$$z = x + yi$$

$$\frac{1}{z} = \frac{1}{x + yi} = \frac{x - yi}{(x + yi)(x - yi)} = \frac{x - yi}{x^2 + y^2} =$$

$$= \frac{x}{x^2 + y^2} - \frac{yi}{x^2 + y^2}$$

$$\operatorname{Re} z = \frac{x}{x^2 + y^2}$$

$$\operatorname{Im} z = \frac{-y}{x^2 + y^2}$$