

$$e^{2z} = 2i$$

$$e^{2(x+iy)} = 2i$$

$$e^{2x+2yi} = 2i$$

$$e^{2x} (\cos 2y + i \sin 2y) = 2i$$

$$e^{2x} \cos 2y + i e^{2x} \sin 2y = 2i$$

$$e^{2x} \cos 2y = 0 \quad e^{2x} \sin 2y = 2$$

$$e^{2x} > 0 \text{ więc } \cos 2y = 0$$

$$\cos 2y = 0$$

$$2y = \frac{\pi}{2} + k\pi$$

$$y = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$y = \frac{\pi}{4} + \frac{2k\pi}{4}$$

$$y = \frac{\pi}{4} + k\pi$$

$$x = \ln \sqrt{2}$$

$$z = \ln \sqrt{2} + i \left(\frac{\pi}{4} + k\pi \right)$$

$$e^{2x} \cdot \sin \left(\frac{\pi}{2} + \frac{k\pi}{2} \right) = 2$$

$$e^{2x} \sin \left(\frac{\pi}{2} + k\pi \right) = 2$$

Wtedy k parzysty:

$$\sin \left(\frac{\pi}{2} + 2k\pi \right) = 1$$

$$e^{2x} = 2$$

$$e^{2x} = e^{\ln 2}$$

$$2x = \ln 2$$

$$x = \frac{1}{2} \ln 2 = \ln \sqrt{2}$$

Wtedy k nieparzysty:

$$\sin \left(\frac{\pi}{2} + (2k+1)\pi \right) = -1$$

$$e^{2x} \cdot (-1) = 2$$

$$e^{2x} = -2 \quad \text{nie ma}$$