

$$|e^z| = e^{\operatorname{Re} z}, \quad \operatorname{arg} e^z = \operatorname{Im} z + 2k\pi$$

$$|e^z| = |e^{x+iy}| = |e^x(\cos y + i\sin y)| = |e^x \cos y + ie^x \sin y|$$

$$= \sqrt{(e^x \sin y)^2 + (e^x \cos y)^2} = \sqrt{e^{2x} \sin^2 y + e^{2x} \cos^2 y}$$

$$= \sqrt{e^{2x} (\sin^2 y + \cos^2 y)} = e^x = e^{\operatorname{Re} z}$$

$$\operatorname{arg} e^z = \operatorname{arg}[e^x(\cos y + i\sin y)] = y = \operatorname{Im} z + 2k\pi$$