$f(x) = \frac{x^2-6x+13}{x-3}$

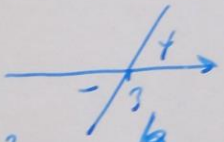
$D_f: \ x \in \mathbb{R} \setminus \{3\}$

Asymptoty:

$\lim\limits_{x\to+\infty} \frac{x^2-6x+13}{x-3} = \frac{x(x-6+\overset{\to 0}{\frac{13}{x}})}{x(1-\underset{\to 0}{\frac{3}{x}})} = +\infty$

$\lim\limits_{x\to-\infty} \frac{x^2-6x+13}{x-3} = -\infty$

Brak asymptot poziomych.



$\lim\limits_{x\to 3^-} \frac{x^2-6x+13}{x-3} = \frac{3^2-6\cdot 3+13}{[0]^-} = \frac{4}{[0]^-} = -\infty$

$\lim\limits_{x\to 3^+} \frac{x^2-6x+13}{x-3} = \frac{9-18+13}{[0]^+} = \frac{4}{[0]^+} = +\infty$

asymptota pionowa $x=3$

asymptota ukośna $y=ax+b$

$a = \lim\limits_{x\to\pm\infty} \frac{f(x)}{x} = \lim\limits_{x\to\pm\infty} \frac{x^2-6x+13}{x^2-3x} = \lim\limits_{x\to\pm\infty} \frac{\cancel{x^2}(1-\overset{\to 0}{\frac{6}{x}}+\overset{\to 0}{\frac{13}{x^2}})}{\cancel{x^2}(1-\underset{\to 0}{\frac{3}{x}})}$

$= 1 \qquad a=1$

$b = \lim\limits_{x\to\pm\infty} (f(x)-ax) = \lim\limits_{x\to\pm\infty} \left(\frac{x^2-6x+13}{x-3} - 1x\right) =$

$$= \lim_{x \to \pm\infty} \left[\frac{x^2 - 6x + 13}{x-3} - \frac{x(x-3)}{x-3} \right] =$$

$$= \lim_{x \to \mp\infty} \left[\frac{x^2 - 6x + 13}{x-3} - \frac{x^2 - 3x}{x-3} \right] =$$

$$= \lim_{x \to \mp\infty} \left[\frac{\cancel{x^2} - 6x + 13 - \cancel{x^2} + 3x}{x-3} \right] =$$

$$= \lim_{x \to \mp\infty} \left[\frac{-3x + 13}{x-3} \right] = -3$$

$b = -3 \qquad y = ax + b \qquad a = 1 \qquad b = -3$

asymptota ukośna: $y = \cancel{x} - 3$

Ekstrema:

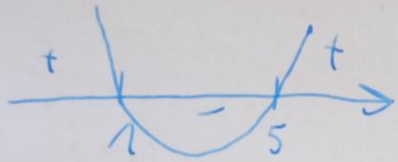
$$f'(x) = \left(\frac{x^2 - 6x + 13}{x-3} \right)' = \frac{(x^2 - 6x + 13)' \cdot (x-3) - (x-3)' (x^2 - 6x + 13)}{(x-3)^2} =$$

$$= \frac{(2x-6)(x-3) - (x^2 - 6x + 13)}{(x-3)^2} = \frac{2x^2 - 6x - 6x + 18 - x^2 + 6x - 13}{(x-3)^2} =$$

$$= \frac{x^2 - 6x + 5}{(x-3)^2} \qquad f'(x) = 0 \iff x^2 - 6x + 5 = 0$$

$\Delta = 36 - 20 = 16 \quad \sqrt{\Delta} = 4$

$x_1 = \frac{6-4}{2} = 1 \quad x_2 = \frac{6+4}{2} = 5$



$$f_{max} = f(1) = \frac{1-6+13}{1-3} = \frac{8}{-2} = -4$$

$$f_{min} = f(5) = \frac{5^2-6\cdot 5+13}{5-3} = \frac{25-30+13}{2} =$$

$$f_{min} = \frac{8}{2} = 4$$