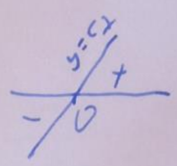
$f(x) = \frac{(x+1)^2}{2x}$ $\qquad D_f: x \in \mathbb{R} \setminus \{0\}$

Asymptoty:

$\lim\limits_{x \to 0^-} \frac{(x+1)^2}{2x} = \frac{(0+1)^2}{[0]^-} = -\infty$



$\lim\limits_{x \to 0^+} f(x) = \frac{(0+1)}{[0]^+} = +\infty$

$x=0$ - asymptota pionowa

$\lim\limits_{x \to -\infty} \frac{(x+1)^2}{2x} = \lim\limits_{x \to -\infty} \frac{x^2(1+\frac{1}{x})^2}{2x} = -\infty$

$\lim\limits_{x \to \infty} \frac{(x+1)^2}{2x} = +\infty$

→ brak asymptoty poziomej.

asymptota ukośna: $y = ax+b$

$\lim\limits_{x \to \pm\infty} \frac{f(x)}{x} = a$

$a = \lim\limits_{x \to \pm\infty} \frac{(x+1)^2}{2x^2} = \frac{\cancel{x^2}(1+\cancel{\frac{1}{x}})^2}{2\cancel{x^2}} = \frac{1}{2}$

$a = \frac{1}{2}$

$b = \lim\limits_{x \to \pm\infty} (f(x) - ax) = \lim\limits_{x \to \pm\infty} \left(\frac{(x+1)^2}{2x} - \frac{1}{2} \cdot x \right) =$

$= \lim\limits_{x \to \pm\infty} \left(\frac{(x+1)^2}{2x} - \frac{x^2}{2x} \right) = \lim\limits_{x \to \pm\infty} \frac{\cancel{x^2}+2x+1-\cancel{x^2}}{2x} =$

$= \lim\limits_{x \to \pm\infty} \frac{2x+1}{2x} = \lim\limits_{x \to \pm\infty} \frac{\cancel{x}(2+\frac{1}{x})}{2\cancel{x}} = 1 \qquad b = 1$

asymptota ukośna $y = \frac{1}{2}x + 1$

Ekstrema

$$f'(x) = \left(\frac{(x+1)^2}{2x}\right)' = \frac{((x+1)^2)' \cdot 2x - (2x)' \cdot (x+1)^2}{4x^2} =$$

$$= \frac{2(x+1) \cdot 2x - 2(x+1)^2}{4x^2} = \frac{4x^2+4x-2(x^2+2x+1)}{4x^2} =$$

$$= \frac{4x^2+4x-2x^2-4x-2}{4x^2} = \frac{2x^2-2}{4x^2}$$

$f'(x)=0 \iff 2x^2-2=0 \quad 2(x^2-1)=0$

$2(x-1)(x+1)=0$

$x=1 \lor x=-1$

$$f_{max} = f(-1) = \frac{(-1+1)^2}{-2} = 0$$

$$f_{min} = f(1) = \frac{(1+1)^2}{2 \cdot 1} = \frac{4}{2} = 2$$

Punkty przegięcia:

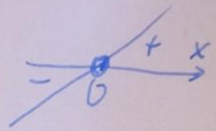
$$y'' = \left(\frac{2x^2-2}{4x^2}\right)' = \frac{(2x^2-2)' \cdot 4x^2 - (4x^2)'(2x^2-2)}{16x^4} =$$

$$= \frac{4x \cdot 4x^2 - 8x(2x^2-2)}{16x^4} = \frac{\cancel{16x^3} - \cancel{16x^3} + 16x}{16x^4} =$$

$$= \frac{\cancel{16}x}{\cancel{16}x^4} = \frac{1}{x^3}$$

$f''(x) = \frac{1}{x^3}$ — $f''(x) \neq 0$ dla $x \in D_f$

brak punktów przegięcia..



x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	(1)	$(1,+\infty)$
y''	$-$	$-$	$-$	X	$+$	$+$	$+$
y'	$+$	0	$-$	X	$-$	0	$+$
y	$-\infty \nearrow$	0	$\searrow -\infty$	X	$+\infty \searrow$	2	$\nearrow +\infty$

$y = \frac{(x+1)^2}{2x}$