$f(x) = \frac{x}{\sqrt[3]{(x-2)^2}}$ $\qquad$ $m_0: x = 0$

$D_f: (x-2)^2 \neq 0$

$x \neq 2$

Asymptoty

$$\lim_{x \to -\infty} \frac{x}{\sqrt[3]{(x-2)^2}} = \lim_{x \to -\infty} \frac{\sqrt[3]{x^3}}{\sqrt[3]{x^2\left(1-\frac{2}{x}\right)^2}} =$$

$$= \lim_{x \to -\infty} \frac{\sqrt[3]{x^3}}{\sqrt[3]{x^2}\,\sqrt[3]{\left(1-\underbrace{\tfrac{2}{x}}_{\to 0}\right)^2}} = \frac{\sqrt[3]{x}}{\sqrt[3]{1-\underbrace{\tfrac{2}{x}}_{\to 0}}} = \frac{-\infty}{1} =$$

$= -\infty$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\sqrt[3]{x}}{1} = +\infty$$

$$\lim_{x \to 2^-} f(x) = \frac{2}{\sqrt[3]{(2-2)^2}} = \frac{2}{[0]^+} = +\infty$$

$$\lim_{x \to 2^+} f(x) = +\infty$$

$x = 2$ - asymptota pionowa

$$= \frac{1}{(x-2)^{\frac{2}{3}}} - \frac{\frac{2}{3}x}{(x-2)^{\frac{5}{3}}} = \frac{(x-2)-\frac{2}{3}x}{(x-2)^{\frac{5}{3}}} =$$

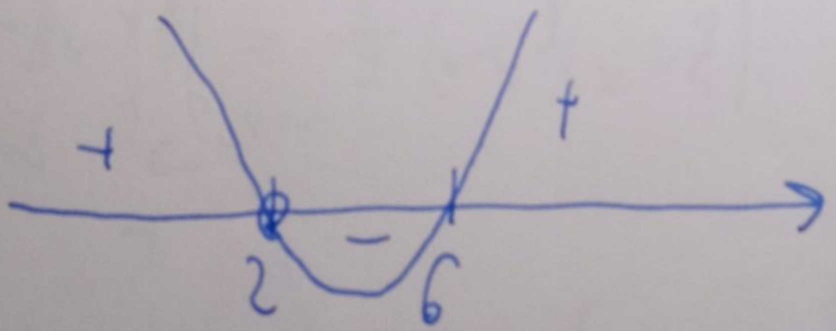
$$= \frac{\frac{1}{3}x-2}{(x-2)^{\frac{5}{3}}}$$

$$f'(x) = 0 \iff \frac{1}{3}x - 2 = 0$$

$$\frac{1}{3}x = 2$$

$$x = 6$$

Znak pochodnej:



dla $x = 6$ f min

$$f_{min} = f(6) = \frac{6}{\sqrt[3]{(6-2)^2}} = \frac{6}{\sqrt[3]{4^2}} =$$

$$= \frac{6}{\sqrt[3]{16}} = \frac{6}{2\sqrt[3]{2}} = \frac{3}{\sqrt[3]{2}} = \frac{3\sqrt[3]{4}}{2}$$

$$f''(x) = \left(\left(\tfrac{1}{3}x-2\right)(x-2)^{-\frac{5}{3}}\right)' = \left(\tfrac{1}{3}x-2\right)'\cdot(x-2)^{-\frac{5}{3}} +$$

$$+\left((x-2)^{-\frac{5}{3}}\right)'\cdot\left(\tfrac{1}{3}x-2\right) = \tfrac{1}{3}(x-2)^{-\frac{5}{3}} - \tfrac{5}{3}(x-2)^{-\frac{8}{3}}\left(\tfrac{1}{3}x-2\right) =$$

$$= \frac{\frac{1}{3}}{(x-2)^{\frac{5}{3}}} - \frac{\frac{5}{3}\left(\frac{1}{3}x-2\right)}{(x-2)^{\frac{8}{3}}} = \frac{\frac{1}{3}(x-2) - \frac{5}{9}x + \frac{5}{6}}{(x-2)^{\frac{8}{3}}} =$$

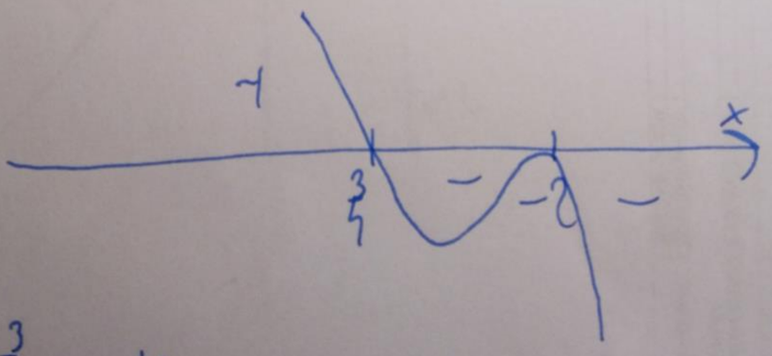
$$= \frac{\frac{1}{3}x - \frac{2}{3} - \frac{5}{9}x + \frac{5}{6}}{(x-2)^{\frac{8}{3}}} = \frac{-\frac{2}{9}x + \frac{1}{6}}{(x-2)^{\frac{8}{3}}}$$

$$f''(x) = 0 \iff -\tfrac{2}{9}x + \tfrac{1}{6} = 0 \qquad -\tfrac{2}{9}x = -\tfrac{1}{6} \quad / \cdot \left(-\tfrac{9}{2}\right)$$

$$x = -\tfrac{1}{6}\cdot\left(-\tfrac{9}{2}\right) = \tfrac{3}{4}$$

Znak drugiej pochodnej $f''(x)$ zależy od iloczynu

$$(x-2)^{\frac{8}{3}}\cdot\left(-\tfrac{2}{9}x + \tfrac{1}{6}\right) = -\tfrac{2}{9}(x-2)^{\frac{8}{3}}\left(x - \tfrac{3}{4}\right)$$



$x = \frac{3}{4}$ – punkt przegięcia

3

x	$(-\infty, \frac{3}{4})$	$\frac{3}{4}$	$(\frac{3}{4}, 2)$	2	$(2, 6)$	6	$(6, +\infty)$
y''	$+$	0	$-$	$\times$	$-$	$-$	$-$
y'	$+$	$+$	$+$	$\times$	$-$	0	$+$
y	$-\infty \nearrow$	$\nearrow$	$\nearrow +\infty$	$\times$	$+\infty \searrow$	min $\frac{3\sqrt{5}}{2}$	$\nearrow +\infty$

pkt przegięcia

asymptota $x = 2$